MCMC Algorithms

Sampling according to a distribution defined analytically

Acceptance-rejection method

Use an **instrumental distribution** g (which we know how to sample from) \Rightarrow to sample from the target distribution f

The general principle is to **choose** g **close to** f and to propose samples from g, to accept some and reject others to get a sample following f.

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Let f be the targeted density function Let g be a proposal density function (from which one knows how to sample) such that, for all x: $f(x) \le Mg(x)$ While $i \le n$: 1 Sample $x_i \sim g$ and $u_i \sim \mathscr{U}_{[0,1]}$ 2 If $u_i \le \frac{f(x_i)}{Mg(x_i)}$, **accept** the draw: $y_i := x_i$ else **reject** it and return to 1.

$$\Rightarrow (y_1, \dots, y_n) \stackrel{iid}{\sim} f$$

Intro 0000000 Direct sampling

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Acceptance-rejection: importance of the proposal



Example of a proposal and a target ditribution for the accept-reject algorithm

Remark: The smaller *M*, the greater acceptance rate

 \Rightarrow the more the algorithm is efficient at sampling from f (less iterations for a sample size n)

So one wishes g the as close as possible to f !

 $\underline{\land} g$ will necessarily have heavier tail than the target

⇒ when the number of parameters increases, acceptance rate decrease svery rapidly (curse of dimension)

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