Bayesian modeling

Bayes in biomedical research I



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This model assumes there is a "true" distribution of Y characterized by the "true" value of the parameter θ^*

$$\hat{ heta}$$
 ?

Historical motivating example

Laplace

What is the probability of birth of girls rather than boys ?

 \Rightarrow observations: births observed in Paris between 1745 and 1770 (241,945 girls & 251,527 boys)

When a child is born, is it equally likely to be a girl or a boy ?

1 the question

2 the sampling model

3 the prior

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The first step in building a model is always to identify the question you want to answer

2 the sampling model

3 the prior

Three building blocks

1 the question

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2 the sampling model

Which **observations** are available to inform our response to this ? How can they be **described**?

3 the prior



Construction of a Bayesian model

Three building blocks

1 the question

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2 the sampling model

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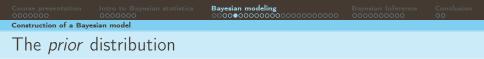
3 the prior

A probability distribution on the parameters θ of the sampling model



- y: the observations available
- ⇒ (parametric) **probabilistic model** underlying their **generation**:

 $Y_i \stackrel{iid}{\sim} f(y|\theta)$



In Bayesian modeling, compared to frequentist modeling, we add a **probability distribution** on the **parameters** $\boldsymbol{\theta}$

 $\theta \sim \pi(\theta)$ $Y_i | \theta \stackrel{iid}{\sim} f(y|\theta)$

 θ will thus be treated like a random variable, but which is never observed !

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Conclusion 00

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Back to Laplace's historical example

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