Priors: pros & cons

Having a *prior* distribution:

- 😁 brings **flexibility**
- 😁 allows to incorporate **external knowledge**
- ☹️ adds intrinsic **subjectivity**

⇒ choice (or elicitation) of a *prior* distribution is sensitive !

Prior properties

1. *posterior* support must be included in the support of the *prior*:
   if $\pi(\theta) = 0$, then $p(\theta|\boldsymbol{y}) = 0$

2. independence of the different parameters *a priori*

Prior Elicitation

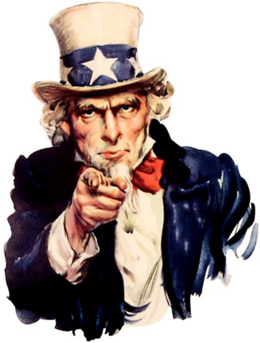
Strategies to communicate with non-statistical experts

⇒ transform their **knowledge** into *prior* **distribution**

- **histogram method**: experts give weights to ranges of values
  ⚠ might give a zero *prior* for plausible parameter values
- choose a **parametric family** of distributions $p(\theta|\eta)$ in **agreement with what the experts think** (e.g. for quantiles or moments)
  (solves the support problem but the parametric family has a big impact)
- elicit *priors* from the **literature**
- ...

SHELF: a tool for prior elicitation from expert knowledge

Your turn !

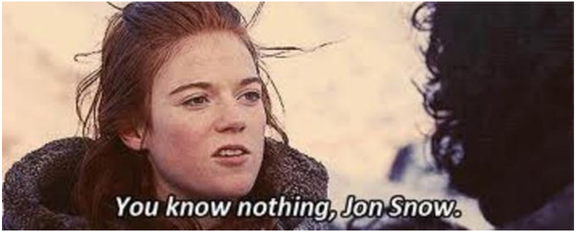


Practicals: exercise 1

The quest for non-informative *priors*

Sometimes, one has **no prior knowledge whatsoever**

Which *prior* distribution to use ?



The quest for non-informative *priors*

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$\Rightarrow$ the Uniform distribution, a **non-informative prior** ?

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2 major difficulties:

1. **Improper distributions** $\int_{\Theta} \pi(\theta) \mathrm{d}\theta = \infty$
2. **Non-invariant distributions**

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2 major difficulties:

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Other solutions ?

Jeffreys' *priors*

A **weakly informative** *prior* invariant through re-parameterization

- unidimensional Jeffreys' *prior*:

$$\pi(\theta) \propto \sqrt{I(\theta)} \qquad \text{where } I \text{ is Fisher's information matrix}$$

- multidimensional Jeffreys' *prior*:

$$\pi(\theta) \propto \sqrt{|I(\theta)|}$$

In practice, parameter are considered independent *a priori*